

## Invited Lecture

### Fostering Student Agency in learning Mathematics: Perspectives from Expert Teachers in Shanghai

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**ABSTRACT** Fostering student agency means developing students' willingness and ability to engage in their own learning. This chapter presents the views of 21 expert elementary school mathematics teachers in Shanghai on fostering student agency. Interview data show that all of the expert teachers value the importance of getting students take ownership of learning and they believe that teachers can contribute significantly in developing student agency. When described the essential features of a classroom where students exercise agency, in addition to focusing on creating an environment that supports student to take up space and actively engage in learning, the expert teachers in Shanghai placed special emphasis on achieving satisfactory learning outcomes. When sharing their strategies for fostering student agency, they commonly mentioned the importance of teachers as role models for their students, which has been less addressed in the literature.

*Keywords:* Student agency; Expert teachers; Teacher's role; Primary school mathematics teaching.

#### 1. Introduction

In China from very early times, we have a favorable attitude towards tradition, authority, official rank and self-cultivation, which are still exerting great influence upon people's thinking and behavior (Li and Chen, 1996). A reflection of the favorable attitude towards authority in education is the high respect for teachers' authority. Chinese teachers control a lesson primarily through prepared instructional tasks, lectures, and frequent exchanges of teacher questions and student responses, which have been reported in the research literature (e.g., Leung, 1995; Mok, 2006). At the same time, Chinese are well aware that learners must take responsibility for their learning. It is a common belief in China that all children can learn and succeed, but

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slower students must devote more time and effort than their peers (Li, 1995). In a study of examining Chinese adolescents' goals and sense of agency, Jin Li, found that her 259 participants from grades 7-12 of middle and high schools in China showed many more effort-based personal agency than social agency in their responses, and the learning virtues such as diligence, concentration, and self-generated activities such as do homework and read books outside classroom were the most often expressed. She called for an emphasis on developing student social agency, such as working with peers or seeking help from teachers (Li, 2006).

Fostering student agency in learning mathematics has been repeatedly emphasized in curriculum reforms in China since the 1990s. The Shanghai Municipal Education Commission issued the Action Plan for School Mathematics Education into the 21<sup>st</sup> Century in 1997 and it clearly indicated that "students are the masters of their own learning. They are the internal factors that affect the learning, while teachers and textbooks are the external factors. Teachers teach so that in the end teachers do not have to teach. Students' development depends to a large extent on their stronger willingness and ability to take responsibility and participate in the learning process" (The Action Plan for School Mathematics Education into the 21<sup>st</sup> Century project group, 1997, p.8). In 2001, the Ministry of Education of China initiated a national curriculum reform at school level, writing in the Standards of Mathematics Curriculum for compulsory education that "students themselves are the masters of their own mathematics learning, while teachers are the organizers, facilitators and collaborators of students' mathematics learning" (Ministry of Education of China, 2001, p. 2). This statement appeared again in its 2011 Curriculum Standards, but with some further suggestions and clarifications: "teachers should play a leading role, handle the relationship properly between teacher direct instruction and student learning agency, and guide students to think independently, explore actively, and cooperate and communicate with their peers" (Ministry of Education of China, 2011, p. 3).

## **2. Student Learning Agency**

Fostering student learning agency is about developing students' willingness and capacity to engage in mathematical learning (Schoenfeld, 2018). It is both a learning goal and a learning process (OECD, 2019). As we conducted the literature review, we found that there are various ways to connect research to the theme of learner agency. For example, from the perspective of educational purpose, the OECD (2019) put forward the Student Agency for 2030 Project, which emphasized that "learn how to learn" is an invaluable skill for every active, responsible and engaged citizen. Identifying culturally appropriate approaches to foster student agency is a challenge for educators in every country.

The emphasis on student learning agency is a reflection of the shift from teacher-centered to learner-centered teaching in curriculum reform. The metaphor of the

teacher as a facilitator of student learning is sometimes falsely taken to mean that the teacher's role is passive, but Boaler (2003) showed us how a school teacher enabled students to work productively with open problems by establishing an environment in which they could engage in talking about their own thoughts and responding to the ideas of their classmates. Sherin (2002) referred to such a classroom environment as a discourse community and gave us more details of how a middle school teacher worked through the school year tried to find his own way to maintain a balance between a student-centered mathematical discourse process and focusing the discussions on significant mathematical content. Based on the social constructivist stance that learning is essentially a social phenomenon, Walshaw and Anthony (2008) considered mathematical discourse involving explanation, argumentation and defense of mathematical ideas as a defining feature of a quality classroom experience.

There are some essential features of high quality teaching, but creating a productive and engaging environment for learning mathematics has become one of them (Middleton et al., 2017; Schoenfeld, 2018). Schoenfeld and his colleagues have carefully constructed a framework for Teaching for Robust Understanding in Mathematics (TRU) to characterize the kinds of teaching that result in students being knowledgeable, flexible, and resourceful thinkers and problem solvers, and "agency, ownership, and identity" is one of the five dimensions of this framework (Schoenfeld, 2018). This particular dimension is used to examine the extent to which students have the opportunity to present their own mathematical ideas and develop their collective understandings through classroom discourse.

Establishing and maintaining a caring and trusting relations with students so that they can learn in a comfortable climate is a favorable condition for high quality teaching. Noddings (2012) agreed that the teacher-student relation is not an equal relation by nature, but she suggested that listening to students' ideas, understanding their needs, responding with care, and integrating moral education with academic learning all contribute to the establishment and maintenance of caring. In an ethnographic study, Noblit (1993) described the story of Pam, an African American elementary school teacher who used her power to construct a caring, safe and engaging learning environment for her students. Pam's class had many routines and rituals to build collective responsibility, and the strong collectivity made each child stronger as a consequence.

Based on our decades of experience working with mathematics teachers in Shanghai, we believe that after more than 20 years of practice and exploration, expert teachers have gained personal insight into developing student agency. Their wisdom needs to be presented in a holistic manner with research that addresses the practices of groups of teachers. To this end, this study targeted 21 expert elementary school mathematics teachers in Shanghai to understand their views and practices on fostering student learning agency. Specifically, we have the following two research questions:

RQ1: What does a classroom where students exercise agency look like?

RQ2: How can teachers effectively foster student learning agency? In particular, what are the specific roles that teachers play? What teaching strategies do they use?

### 3. Methods

#### 3.1. Data collection

This study is primarily based on interview data that we collected in 2019. In addition to interviewing these 21 teachers, we also observed and recorded or collected teaching videos from eight of them to help us better understand their views and practices that they mentioned in the interviews. On the other side, we interviewed a total of 18 students from 3 elementary schools. We also asked 38 fifth-grade students to anonymously write down one strength and one possible weakness that impressed them most after an open lesson given by an expert teacher who participated in this study. However, only data collected from teachers were used in this report, and our conclusions were based primarily on the interviews with the 21 expert teachers. In our interviews, we asked the expert teachers to share their perspectives **and** stories on the following questions:

1. How to interpret that “teachers should play a leading role in the teaching process and students are the masters of their own mathematics learning”?
2. How teachers can use questions to assist students exercise their learning agency?
3. Give a picture of a classroom where students exercise agency in learning; and
4. for the subtraction question  $50 - 26 = \underline{\quad}$ , what would you do next if you see that the students have answers 24 and 34?

The teachers were asked to share stories or concrete examples because we wanted to understand their own interpretations of the theoretical views advocated by the official curriculum standards and to learn more about their unique teaching strategies from their stories.

We interviewed with 17 expert mathematics teachers from 15 primary schools in Shanghai and 4 expert teacher educators who used to be primary school teachers in Shanghai but mainly do in-service mathematics teacher training now. They are named as “expert” because 4 of them are outstanding teachers in Shanghai with the rank of professor level (the top professional title to school teachers in China) and the others are senior level teachers (the second high). Seven of them are also called Master (TEJI) Teacher (an honour to recognize outstanding school teachers and principals, but it is not a professional rank. In this study, we coded these 7 Master Teachers as A1–A7 and the other 14 expert teachers as B1–B14). Ten of them were males and 11 were females. Sixteen of them had been teaching for 20 years or more, with the least being 11 years

and the most being 41 years. Four of them had not taught Grades 1 and 2, the remaining 17 teachers had taught all grades of elementary school.

Teachers were interviewed by telephone, email, or in person. The main reason for the inconsistent interview format is that very few mathematics teachers in each school have the professional ranks we set, so our participants came from 18 workplaces, which are located in various regions of Shanghai. Also, they were all busy, so it was more convenient for them to be interviewed by phone or email. If we felt that something was not clear in the written responses, we asked the interviewees for further clarification by phone or in person. It seems that the telephone and in-person interviews may have been more detailed and in-depth in some areas due to the interaction, whereas the teachers interviewed by email may have been more comfortable in providing us with details of the relevant learning tasks and implementation processes. All interviews were conducted focusing on the 4 interview questions. Both the in-person and telephone interviews were audio recorded.

### **3.2. Data analysis**

After completing the interviews, we transcribed all audio-recorded interview data into text to facilitate subsequent coding. Data coding and analysis for this study was conducted in the original language of Chinese. Selected data was translated into English to provide evidence for the results of this study. Based on the purpose and the research questions of this study, we conducted a qualitative content analysis to describe and identify themes in the participants' responses. Considering that fostering student agency is culturally influenced, we decided to use an inductive approach, which means that codes, categories, or themes were extracted directly from the data (Cho and Lee, 2014). Firstly, the three authors recorded then summarized independently the interview texts of seven teachers they were responsible for, sentence by sentence and example by example, on worksheets. The worksheets were sent to a second coder for cross-checking. The second coder marked the descriptive labels with which he/she disagreed, added a brief comment, and sent the worksheets back to the first coder. If not all comments were accepted, they would discuss the labels by phone to reach an agreement. Then, each of us took the lead in coding the data for 1–2 interview questions. By repeatedly reading the worksheets and sometimes looking back at the interview transcripts or watching the teaching videos, some key words and ideas emerged and were noted in a list of key terms or short sentences. We categorized all teachers' responses to each interview question with a set of key terms or short sentences. Data coding was cross-checked again, however, this time all three authors checked the coding for each interview question. We organized several video conferences to resolve the coding inconsistencies and fix the problems we encountered in categorization. We then drafted a summary report outlining the preliminary findings for each interview question. In the final stage of data coding, all interview data were categorized according to the research questions of this study and the themes underlying the teachers'

responses to each research question were identified. The first author went through all the interview transcripts again, fine-tuned and combined a few repetitive codes under the structure of the two research questions and finalized the codes. We did not code the collected teaching videos, but we selected some teaching episodes from the videos to support the findings of this study.

## 4. Results

### 4.1. *Essential features of a classroom where students exercise agency*

Through recursive analysis of the 21 expert teachers' descriptions of a classroom where students exercise agency, four themes were identified and each feature is represented by 2-3 indicators. In general, all of the four features were valued by the expert teachers.

#### 4.1.1. *Full and active participation*

The first feature shows the degree of learners' behavioral engagement and cognitive engagement (Fredricks et al., 2004) in their learning. It includes learners' external actions, such as manipulations, as well as internal actions, such as thinking and working quietly and independently, using their hands, brains and hearts. The first indicator "intellectually engage in hands-on or other activities" highlights that manipulative activities must be accompanied by mathematical thinking. Another indicator, "collaborate and communicate among community members", reflects a general recognition among the expert teachers of the importance of social interaction and communication in mathematics learning. Ms. A2 shared with us a story about her teaching of "Recognizing milliliters and liters".

After the class, a teacher saw me bring back some disposable cups with marks left by students and asked me what they were for. I said: to measure 100ml. The teacher said, "No need, we have 100 ml measuring cups in the school lab, just show them to the children." I told her that I designed an activity called "Play with 100 ml of water" and made 2 requirements: 1). estimate the water level of 100 ml based on 10 ml of water; 2). take the 100 ml of water estimated by your group (use disposable cups only). During the sharing time, one group of students said: "We did it by counting, 10 ml is just a layer on the bottom of the cup, 100 ml is  $10 \times 10$  ml, so I made my mark here." Another group of students immediately raised their hands and said: "This mark is not correct, the water must be more than 100 ml. The cup is smaller at the bottom and it gradually gets bigger from the bottom to the top, so our group thought....."

She ended the story by saying that mathematics is not a subject that depends on experiments, it depends on thinking. Langer-Osuna and Esmonde (2017) also agreed that the integration of thinking and manipulation is crucial for mathematical learning,

noting that “students’ agency is framed in terms of the degrees of freedom they experience in being able to intellectually engage with mathematics” (p.645).

#### 4.1.2. *Taking up space*

This dimension of engaging classrooms also includes two indicators: “discuss questions from students and build on each other’s’ idea” and “involve students in the evaluation process”. It implies that instruction focuses on learners’ learning, students become part of the classroom community, engage in meaningful, critical, and respectful dialogue, and use their capacity to contribute to the development of the collective agency (Bandura, 2000; Hand, 2012). The following keywords were used more often by the expert teachers in their interviews: “have a voice, agency, ownership, student ideas, reflective listening, questions, questioning, add, contribute, build on, solve problems, agree, disagree, justify, persuade, whole class discussion, authority, facilitator, openness, support, self-study, community, self-assessment, peer assessment”. This is a key feature of a classroom where students take ownership of their learning, and as Cobb et al. described, “authority being distributed across the students and the teacher. The students were expected to determine the reasonableness and adequacy of solutions as they presented, listened, and asked questions” (p.54).

In general, Chinese students rarely ask questions in class (Jin and Cortazzi, 1998), developing their willingness and capacity to listen attentively, think critically, and communicate friendly takes years, and responding appropriately to students’ ideas on the spot can be challenging for any teacher; however, the expert teachers who participated in this study generally agreed that it made a great deal of sense to use their authority to support students taking up space and exercising agency in their learning. According to Mr. A6, “whether students have the courage to present their own ideas separates good teachers from average teachers”. Mr. A5 said, “having students ask questions is the key to a good lesson, the key to personalized learning and deep learning”. Explaining why she believes teachers should not be the only source of mathematical ideas and assessments, Ms. B4 said:

If only teachers evaluate the students, their comments are not diverse enough. Involving students in the evaluation process can push them listen to their peers and think reflectively. And, at the end of a lesson, I often invite students to evaluate their performance today. Successful peer assessment make students learn from each other, and make progress together; Self-assessment helps students to see new things they have learned and to ask questions they have, and it promotes their own development.

In contrast to “full and active participation”, the feature of “taking up space” concerns collective agency rather than individual agency in learning.



#### 4.1.3. *A pleasant and supportive learning environment*

It is enjoyable for students to try and explore on their own and eventually discover a pattern or solve a problem, but limited by their age, experience, and knowledge, they usually need the support of their teachers and peers to be successful. The first indicator is “engage in interesting tasks with a certain degree of openness”. The reason why open-ended but interesting problems were recommended is to allow students of all levels to participate. The second indicator is “have adequate time and resources for inquiry”. Ms B7 said in her interview: “If you really want students to explore, you must give them time. Some students won’t succeed if they haven’t reached a certain number of attempts. Maybe after a few more tries, they will find the pattern”. The last indicator is “listen each other carefully and allow sharing of uncomplicated, flawed replies”. Ms. B5 shared with us a story that happened in her class.

I remember we talked about a multiplication equation, there are five rows of flowers, each row has six flowers, so  $6 \times 5$ , a total of thirty flowers. One child said I have another way,  $5 + 5 + 5 + 5 + 5 + 5$ , and the other children snickered. I said seriously, “I think he is very good. .... What’s so funny?” Then I turned to him: “So why do you think the other students reacted that way?” “Because my way is tedious”. “It’s okay, you deserve more praise. First, you know why it needs improvement, and second, you have the courage to voice your idea.”

The importance of attentive listening and mutual respect has been stressed in the literature (Cobb et al., 2009; Middleton et al., 2017; Ministry of Education of China, 2011; Noddings, 2012). Middleton et al. (2017) stated that “engagement varies depending on the level of social risk students feel comfortable taking” (p.687).

#### 4.1.4. *Satisfied learning achievement*

The expert teachers in Shanghai placed special emphasis on achieving satisfactory learning outcomes. Its indicators are “learn with great interest” and “develop conceptual understanding, procedural fluency, proficiencies and good habits”. They involve students’ learning achievement in cognitive, affective, and meta-cognitive aspects. The first indicator is related to emotional engagement (Fredricks et al., 2004) in learning. When describing a classroom where students exercise agency, Mr. A4 said, “the lesson must be engaging. Not only are students happy to learn, but teachers are also enthusiastic to teach. It looks like all of us are enjoying our time in the classroom.” In an evaluation report of the UK-China Mathematics Teacher Exchange Programme, Boylan and his research team described their views as outsiders to Shanghai mathematics: “Shanghai mathematics education is a mastery approach and so is premised on the belief that all pupils can succeed as mathematical learners. Classroom practices and organization of mathematics teaching follow from this belief. Shanghai whole class interactive teaching aims to develop conceptual understanding and procedural fluency.” (Boylan et al., 2016, p.15). The expert teachers we interviewed



believe that it is their responsibility to help students, especially the late-bloomers, grow, even though it may take years to see them succeed.

#### ***4.2. Strategies for fostering student agency***

All 21 expert teachers agreed that teacher can contribute significantly in developing student agency and shared with us some of their own teaching strategies. Mathematics teachers take on four distinct roles as diagnosticians of students' thinking, conductors of classroom discourse, architects of curriculum, and river guides who are flexible in the moments of teaching, which are the "images" of expertise in mathematics teaching identified by Russ et al., (2011), and this can serve as a framework to organize our findings about how teachers foster students' learning agency, but it is important to add one more "image", namely *Ren Shi* (人师, role models for students, in Chinese) presented by Hsieh et al. (2018).

##### *4.2.1. Strategies used by teacher as architect of curriculum*

These expert mathematics teachers in Shanghai value and excel at teaching design. They carefully design teaching goals, tasks, learning processes, and formative assessments prior to class so that students can actively and successfully engage in deep learning. They shared with us a lot of instructional tasks or questions they designed and explained their intentions as well. We found three strategies they use associated with careful teaching design. Due to space limitations, a brief description of each strategy is provided.

*Use appropriate tasks/problems to help students succeed.* Tasks play a critical role in students' learning outcome and learning opportunities (Hiebert and Wearne, 1993). Carefully designed situational task sequences can turn a learners' attention to abstract similarities and develop their conceptual understanding (Margolinas, 2013). Similarly, the Shanghai teachers suggested to use a coherent sequence of tasks/problems to guide students in constructing knowledge on their own, to design tasks with a degree of openness to allow all students to have a voice, and ask questions within "zone of proximal development".

*Use interesting tasks to engage students.* They recommended presenting tasks in real-life or interesting contexts, inviting a group of students to work as a team to teach their classmates like a teacher, inviting students to ask relevant questions based on the given conditions, and solving problems from students.

*Have students try the tasks before class.* They proposed assigning tasks before class so that students come prepared; ensuring enough time for exploration, personal reflection, and discussion of common errors and misconceptions; and having students to learn new content on their own that is similar to a topic they have already learned. "Knowing millimeters and decimeters" was a lesson that Mr. A4 gave to Grade 2 students. In the first part of this lesson, Mr. A4 organized eight activities to help

children visualize 1 mm. They are: recognizing it on a ruler, drawing it on paper, telling the class how they feel about it, finding it in the classroom, making it with two fingers, estimating and measuring the thickness of a book in millimeters, talking about examples of using millimeters as a unit in their lives, and discovering the relationship between millimeters and centimeters with the help of a ruler. He applied the techniques such as “use a coherent sequence of tasks to guide student learning”, “design tasks with a degree of openness to allow all students to have a voice”, “ask questions within ZPD” and “present tasks in real-life or interesting contexts”. In teaching of converting between units of length, he presented the staircase model (the children had seen this model before when they studied numbers) on the blackboard, put a millimeter card on the step, and ask: “1mm up, 1mm up, grow up to 10 mm, it is \_\_\_?” The whole class answered in chorus “1 centimeter”. He continued, “In fact, when we count by ones and reach 10 ones, we go up a step and create a new unit a ten; and when we count by tens and reach 10 tens, we go up another step, a new unit, a hundred. How do you think what we learned before helps us learn today?” With his help, the class successfully mastered the relationship between units of length and discovered a connection between the learning of numbers and the learning of units of length, thus deepened their understanding of base-10 system. He also arranged two engaging exercises at the end for formative assessment, “Who’s Missing?” and “Uncovering the Truth,” which asks children to identify and correct units used incorrectly in a short story. This example illustrates how teachers can effectively involve students in learning and develop their sense of measurement by designing engaging tasks before class.

#### 4.2.2. *Strategies used by teacher as conductor of classroom discourse*

The image of “mathematics teacher as conductor of classroom discourse” refers to teachers develop trusting classroom communities, direct and shape the classroom discourse, and motivate students to build on each other’s’ idea.

*Arrange communication of different sizes.* The Shanghai expert teachers flexibly organize discussions of different sizes for two-person, four-person and the whole class, with particular emphasis on whole-class discussions. The use of two-person exchange allows for greater efficiency. Using whole-class dialogue facilitates the resolution of disputes, demonstrates different ways to solving the same problem, and also allows the teacher to enact a mentoring role, such as intervening early when students are observed to be having learning difficulties.

*Form a pleasant and supportive learning environment.* The expert teachers mentioned techniques such as seeing several students raise their hands to wait a few extra seconds to get more responses from the class; assisting individual students to clarify their thoughts and expressions, and respecting and praising the effort and courage of students who answered inappropriately. Walshaw and Anthony (2008) also highlighted the importance of scaffolding students’ ideas.

*Engage students in productive class dialogue.* In China, due to the large population, developing economy, and collective culture, large classes are common (Jin and Cortazzi, 1998). The expert teachers suggested some techniques to improve student communication, such as forming a rule in mathematics classroom that agree requires a reason and disagree requires a counterexample; inviting different students to explain or comment on a peer's method to show different ideas, inspire each other, and build on each others' ideas; and having students guess what questions the teacher could ask to motivate students to think and learn how to ask questions. Primary school teachers in China often lead the class in applauding a student's wonderful ideas, which has a powerful encouraging effect on students.

#### 4.2.3. *Strategies used by teacher as diagnostician of students' thinking*

Teachers' expertise in subject matter knowledge and pedagogical content knowledge (Ball et al., 2008) enables them to provide children with timely and helpful diagnosis-based guidance and feedback (Fraivillig et al., 1999). These Shanghai mathematics teachers talked about strategies they used both in designing instruction before class and implementing their lesson plans during class.

*Analyze student learning to inform teaching.* Specific techniques that the expert teachers shared with us were: clarifying what students know, what they don't know, what they want to know, and what they can know with the help of others; diagnosing student learning based on their work and the questions they ask; and using students' work, especially their errors and misconceptions, as teaching resources.

*Listen to students and quickly discern their real thoughts.* They suggested that teachers give timely and helpful feedback, especially on what was done well; guide the class to discover where and why they went wrong and how to avoid repeating it; categorize students' various methods mathematically to reveal the nature of these methods that may not look alike; and decide quickly which ideas should be discussed further.

*Assess learning outcomes after intervention.* The Shanghai teachers preferred to arrange an exercise after their intervention. Using the lesson "subtraction with trading" as an example, Mr A5 explained that his "specialized exercise" means that the teacher prepares six to eight subtraction equations and has the class verbally classify them into two groups: with trading or without trading, no further calculations are needed. This is because once students have determined that a subtraction equation requires a trading, it then becomes a familiar task that students can easily complete. Another technique the teachers advocated is inviting students to participate in assessment, including both self-assessment and peer assessment. They agreed with the idea of assessing *with* students, rather than just *to* students. They suggested to have students who answered incorrectly share their new understanding and give their reminders to the class at the end of the class.

#### 4.2.4. *Strategies used by teacher as river guide for learning journey*

Any unexpected responses or contingencies may arise during the learning journey, and teachers are expected to respond quickly and appropriately to them and modify their original lesson plans accordingly. The recommended strategies are:

- *Elicit student responses.* These expert teachers suggested two approaches to break the ice. One is to ask a different question to elicit responses or break down difficulties. The second is to provide manipulatives or draw a diagram to facilitate thinking.
- *Help students in a smart way.* Teachers should not overlook mistakes, but some errors may be caused by carelessness, and students can find and correct them in explanations to the class. In this case, for example, for the equation  $50 - 26 = ?$  teacher could ask the student who answered 34 how to change the equation, then the answer 34 would be correct. Another teaching tip is to postpone teacher comments for those students who are particularly active or too far ahead, and instead listen to other students' ideas and come back to comment later when appropriate.
- *Plan ahead, but be flexible.* Shanghai has unified mathematics textbooks, but designing the teaching process and preparing learning tasks is a daily task for every teacher. The expert teachers advised young teachers not only to be flexible in adjusting their original lesson plans and not to be forced to complete all the prepared tasks, but also to record post-lesson teaching reflections on their lesson plans in order to improve their future teaching.

#### 4.2.5. *Strategies used by teacher as role model for students*

There is an old Chinese saying, "A teacher for a day is a father for a lifetime", referring to the deep feelings between teachers and students. Teachers need to be strict with their students, but also love them, care about their needs, interests, thoughts and emotions, assist them to succeed, and always behave as a role model for students. The expert teachers believed the following two points are important.

*Value the moral development of students.* Moral education should be integrated into the intellectual education of every subject in school. The expert teachers placed special emphasis on: praising good behavior, expressing your expectations and love for your students; establishing teacher authority through excellence in personality and scholarship; and modeling the virtues of seeking the truth, convincing others with reason, and being willing to correct any mistakes.

*Guide students on how to learn.* They emphasized that teachers should develop students' enthusiasm for inquiry, knowing that inquiry begins with observation and a reasonable guess, followed by verification and finally expression. They should assist students in asking questions, stating and explaining their ideas confidently, and being willing to learn from others. They should proactively use new technology platforms or other resources to facilitate students' self-learning. Attention also should be given to developing good learning habits, such as "listening and comparing others' ideas with

one's own," "saying full sentences and using stem sentences (Boylan et al., 2016, p. 71)," "reading textbook and questions carefully and circling key points," "writing neatly and formally," and "using estimation and other ways to check an answer".

## 5. Conclusion

For teachers who intend to foster student agency in learning mathematics, it is crucial to identify the essential characteristics of a classroom in which learners exercise agency. From our interviews with 21 expert elementary school mathematics teachers in Shanghai, we learned that their perspectives on this are as follows: students collaborate with their teacher, participate in learning activities behaviorally, cognitively and affectively. As members of a learning community, they take the initiative to ask questions or contribute ideas, build on others' thinking, and deepen their own understanding through comparison and reflection. Of course, the active engagement cannot be achieved without having a pleasant and supportive learning environment. At the same time, being a good lesson must also result in satisfied learning outcomes, not just be fun for students. Thus, in their view, it is more like a teacher-directed and learner-engaged classroom, where both teacher guidance and student engagement are essential.

Consistent with what is stated in the curriculum standards (Chinese Ministry of Education, 2011), all the expert teachers involved in this study agreed that students themselves are the masters of their own learning and students' learning should be placed at the center of education, but teacher's leading role must be maintained. Eleven teachers explicitly pointed out that student exercise agency is actually a reflection of the teacher's leading role, the more fully the teacher's leading role is enacted, the more motivated and proactive the students will be in their learning. Since achieving satisfactory learning outcomes is a common goal for both teachers and students, teacher guidance and student agency can coexist in harmony. Their teaching stories also illustrate some strategies on how to achieve such coexistence. For example, in the lesson "Knowing millimeters and decimeters" we mentioned earlier, Mr. A4 prepared a series of focused, coherent and engaging learning tasks. He introduced parallel learning activities to visualize 1 millimeter and 1 decimeter, but students took greater ownership in the latter set of learning activities because he believed that the experience of learning 1 millimeter would assist them in learning 1 decimeter on their own. Conversions between units are often difficult for children, and they try to remember them by rote. His solution was to draw a staircase model on the board to make converting length units simple and meaningful. The expert teachers know where to focus their instruction, which learning barriers to break through are critical to success, and what learning tasks could scaffold students' thinking and make their learning easier and productive. Teacher-planned activities and the various mathematical discussions that surround them are critical to learning (Ben, 2016; Walshaw and Anthony, 2008).

If we divide teachers' work in a lesson into four phases: preparation, initiation, development, and closure, then we find that the teachers could play different main roles in each phase. According to the data we collected, teachers seemed to have more

control in the preparation and initiation phases. Once the lesson started, they encouraged their students take an active role and were prepared to intervene as conductors and river guides when appropriate. In all phases, teachers always disciplined themselves with the expectation that they were role models for their students. Teacher's role is multifaceted, professional, and irreplaceable (Russ et al., 2011).

The call to foster student agency in learning requires that student voice be heard, but actually doing so in teaching remains very complex due to the time constraints of a lesson. As a result, the expert teachers participated in this study placed great emphasis on designing and structuring learning tasks, quickly diagnosing student thinking, organizing whole-class dialogue, using new technology platforms to facilitate students' self-learning, and involving students in assessment. It is worth noting that in China, primary school math teachers typically teach only math and will continue to teach the same class for several years. The members of each class are essentially fixed, and they work together as a learning community for all subjects. These are conducive to the development of good learning habits, classroom norms and interpersonal relationships that take years to develop.

Enabling students to take ownership of their learning requires, on the one hand, that teachers have excellent and varied expertise and the courage to explore, change and frequently reflect on their teaching practices; on the other hand, it requires that students have the willingness and ability to engage in active learning. Finally, and most importantly, it requires teachers to have a deep love and commitment to education and to their students. In the interviews, two teachers talked about the same lesson they taught a few years apart and told us how and why their teaching design had changed. It can be said that they have strong teacher agency in their teaching. Building teacher agency is the key to addressing student agency.

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